

Hermite Curves

CS 418

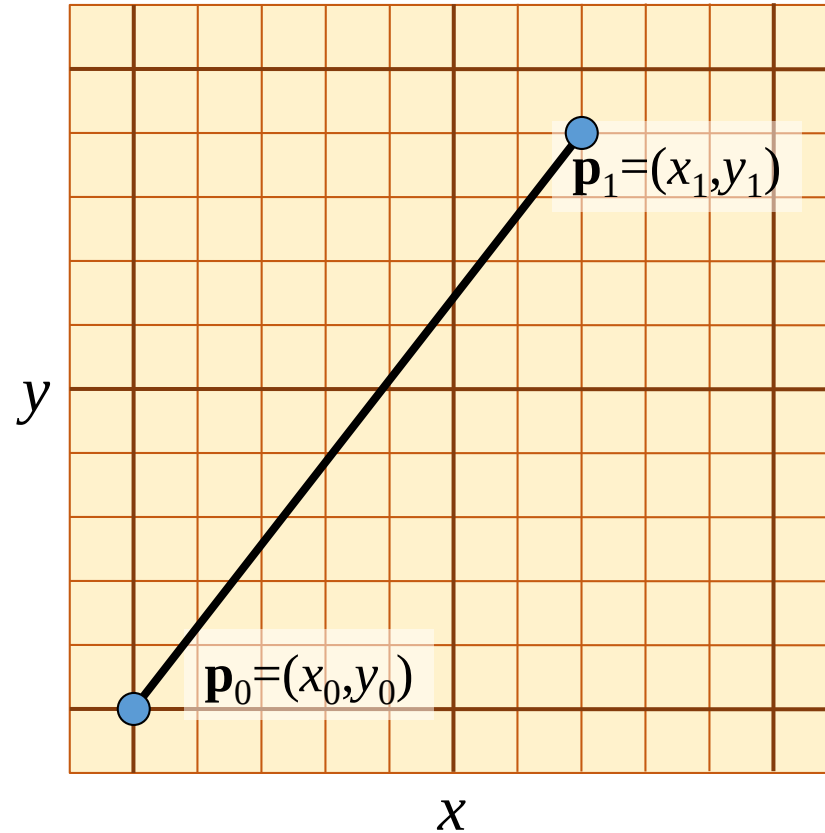
Interactive Computer Graphics

John C. Hart

Linear Interpolation

- Define a parametric function $\mathbf{p}(t)$

$$\mathbf{p}(0) = \mathbf{p}_0, \mathbf{p}(1) = \mathbf{p}_1$$



Linear Interpolation

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$$\mathbf{p}(0) = \mathbf{p}_0, \mathbf{p}(1) = \mathbf{p}_1$$

- Separate into coordinate functions

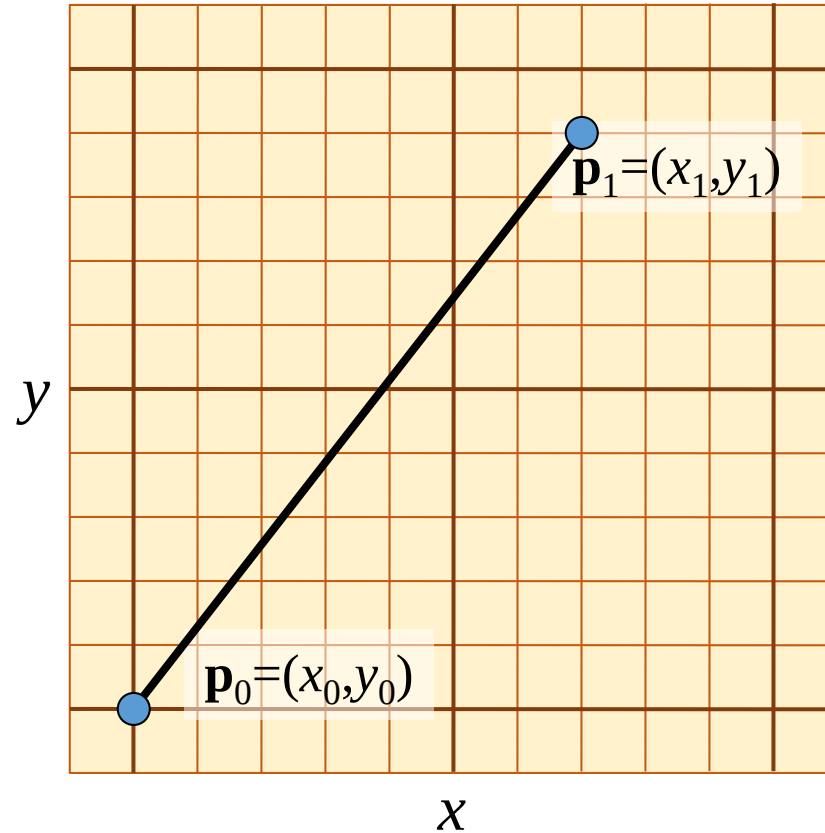
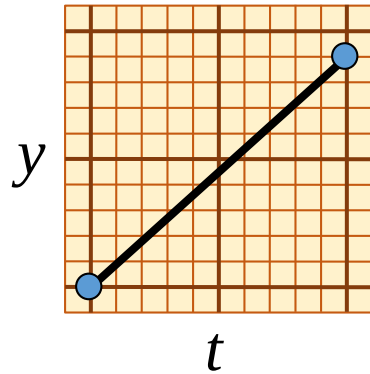
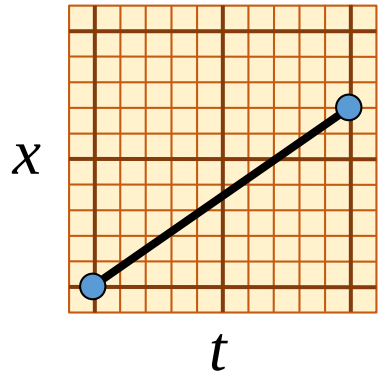
$$\mathbf{p}(t) = (x(t), y(t))$$

$$x(0) = x_0$$

$$y(0) = y_0,$$

$$x(1) = x_1$$

$$y(1) = y_1$$



Linear Interpolation

- Define a parametric function $\mathbf{p}(t)$

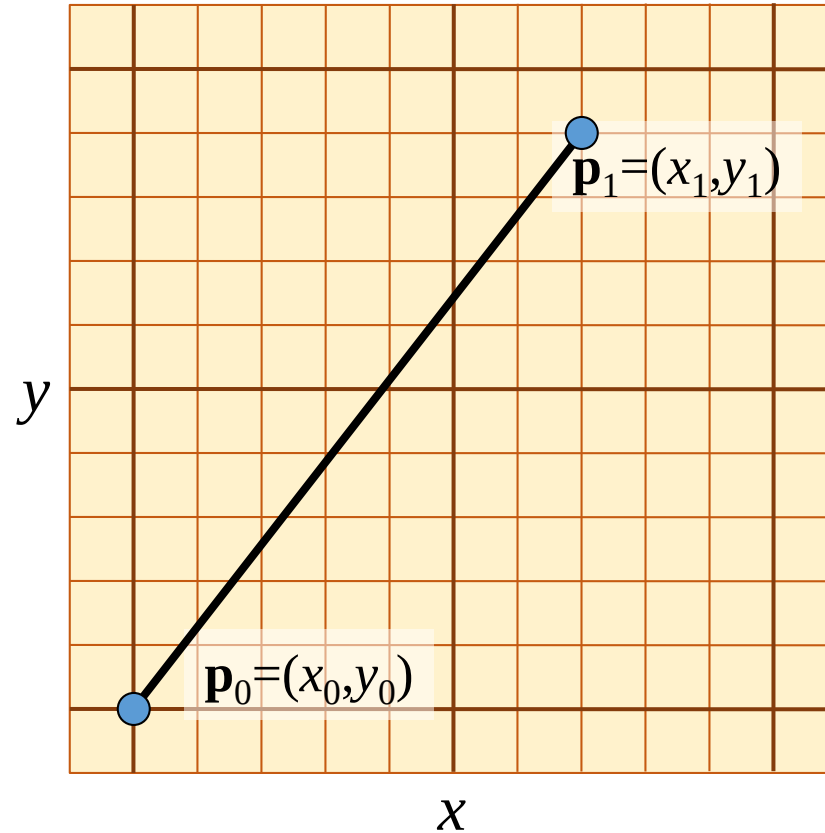
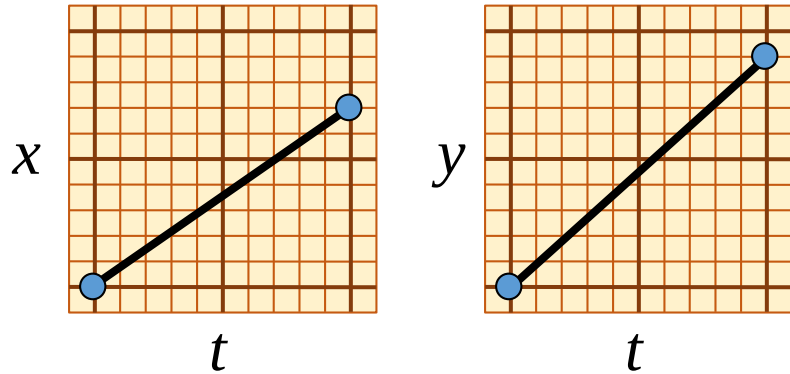
$$\mathbf{p}(0) = \mathbf{p}_0, \mathbf{p}(1) = \mathbf{p}_1$$

- Interpolate

$$\mathbf{p}(t) = \mathbf{p}_0 + t(\mathbf{p}_1 - \mathbf{p}_0) = (1-t)\mathbf{p}_0 + t\mathbf{p}_1$$

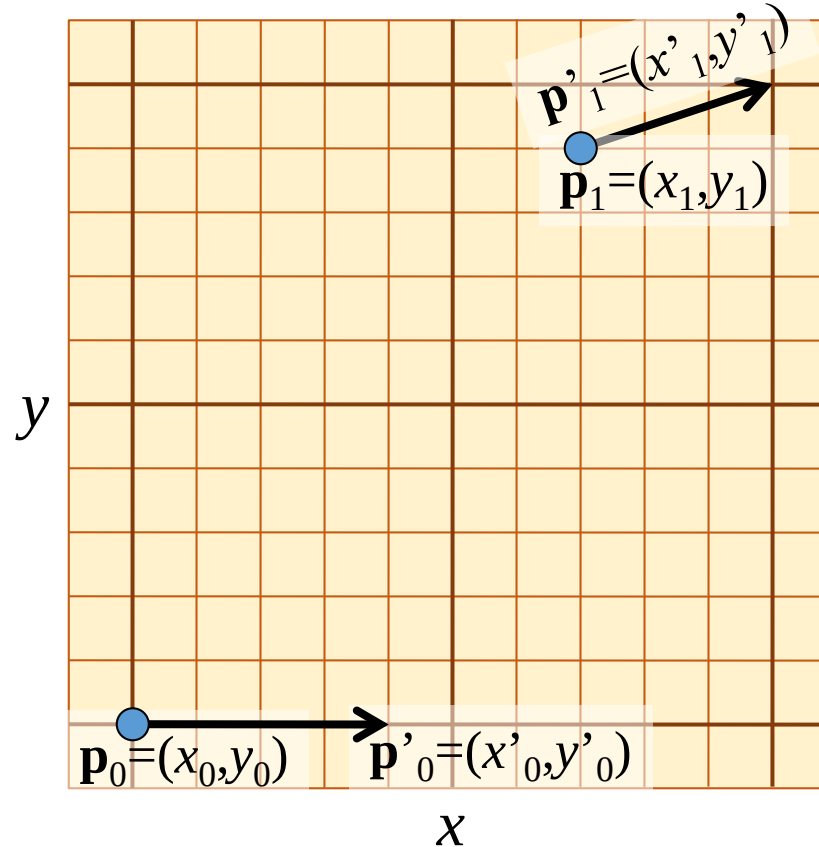
$$x(t) = x_0 + t(x_1 - x_0) = (1-t)x_0 + tx_1$$

$$y(t) = y_0 + t(y_1 - y_0) = (1-t)y_0 + ty_1$$



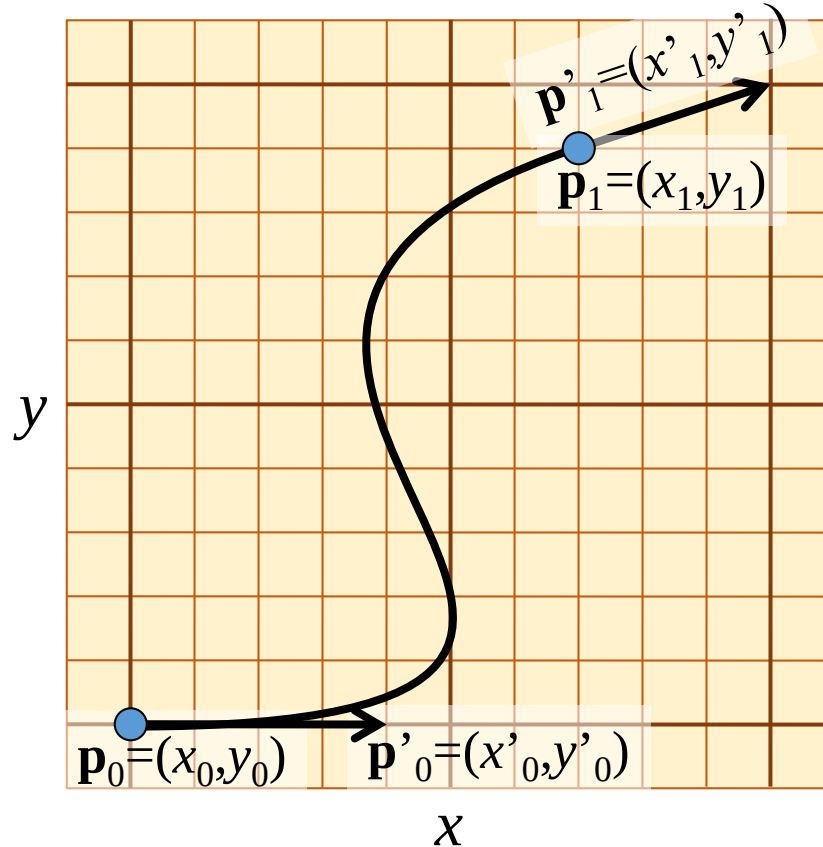
Hermite Interpolation

- From point $\mathbf{p}_0$ along $\mathbf{p}'_0$
to point $\mathbf{p}_1$ toward $\mathbf{p}'_1$



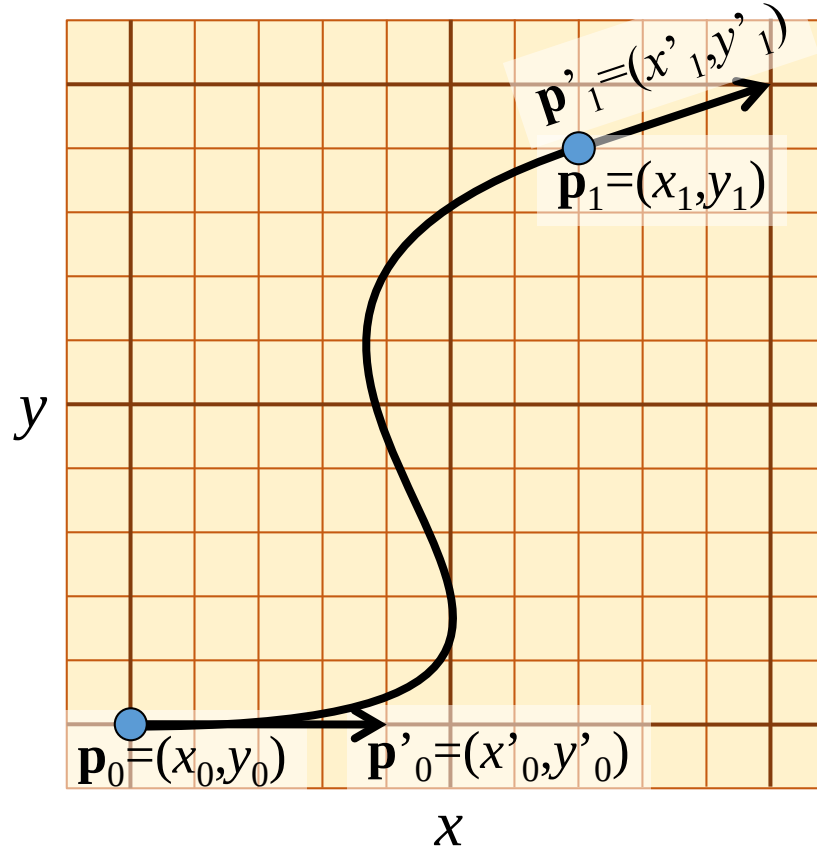
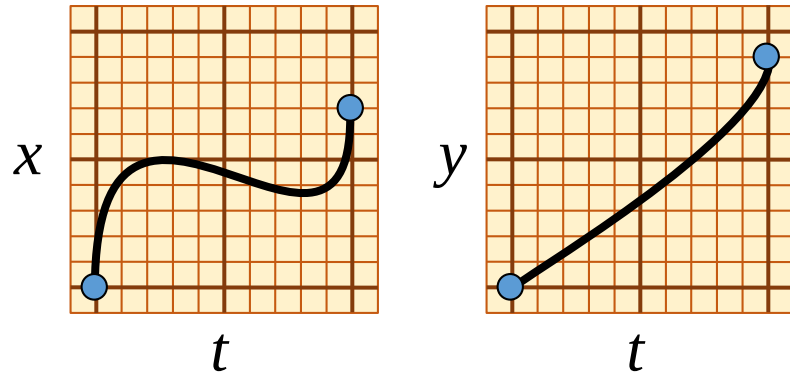
Hermite Interpolation

- From point $\mathbf{p}_0$ along $\mathbf{p}'_0$ to point $\mathbf{p}_1$ toward $\mathbf{p}'_1$
- Define a parametric function $\mathbf{p}(t)$
 $\mathbf{p}(0) = \mathbf{p}_0, \mathbf{p}(1) = \mathbf{p}_1$
 $\mathbf{p}'(0) = \mathbf{p}'_0, \mathbf{p}'(1) = \mathbf{p}'_1$



Hermite Interpolation

- Define a parametric function $\mathbf{p}(t)$
 $\mathbf{p}(0) = \mathbf{p}_0, \mathbf{p}(1) = \mathbf{p}_1$
 $\mathbf{p}'(0) = \mathbf{p}'_0, \mathbf{p}'(1) = \mathbf{p}'_1$
- Separate into coordinate functions
 $x(0) = x_0, x(1) = x_1$
 $x'(0) = x'_0, x'(1) = x'_1$

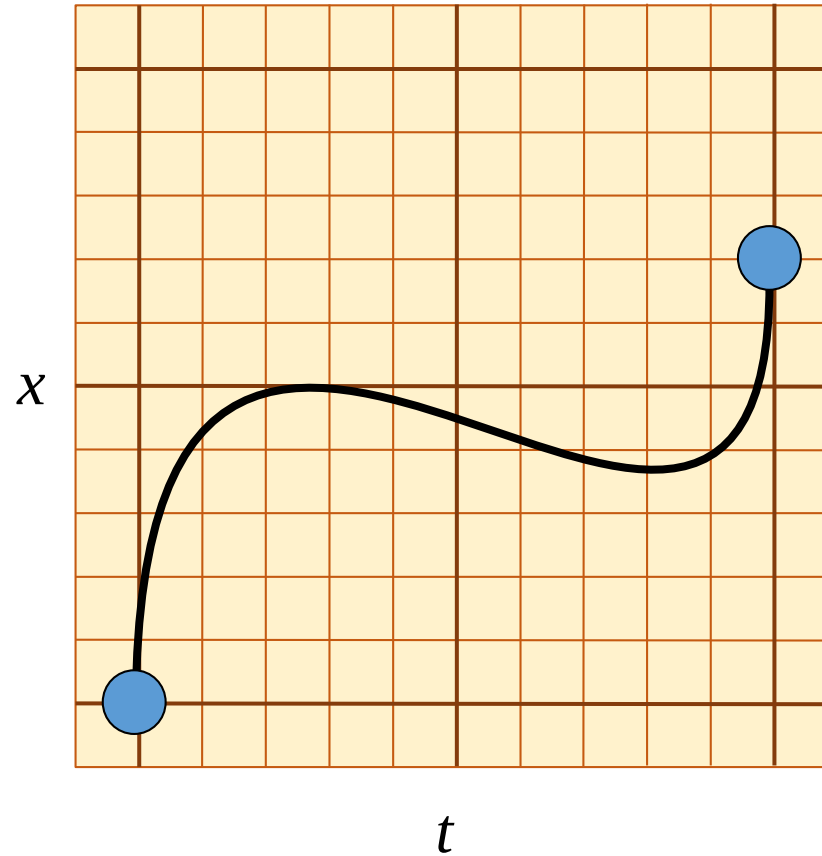


Hermite Interpolation

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$$x'(0) = x'_0, x'(1) = x'_1$$



Hermite Interpolation

- Separate into coordinate functions

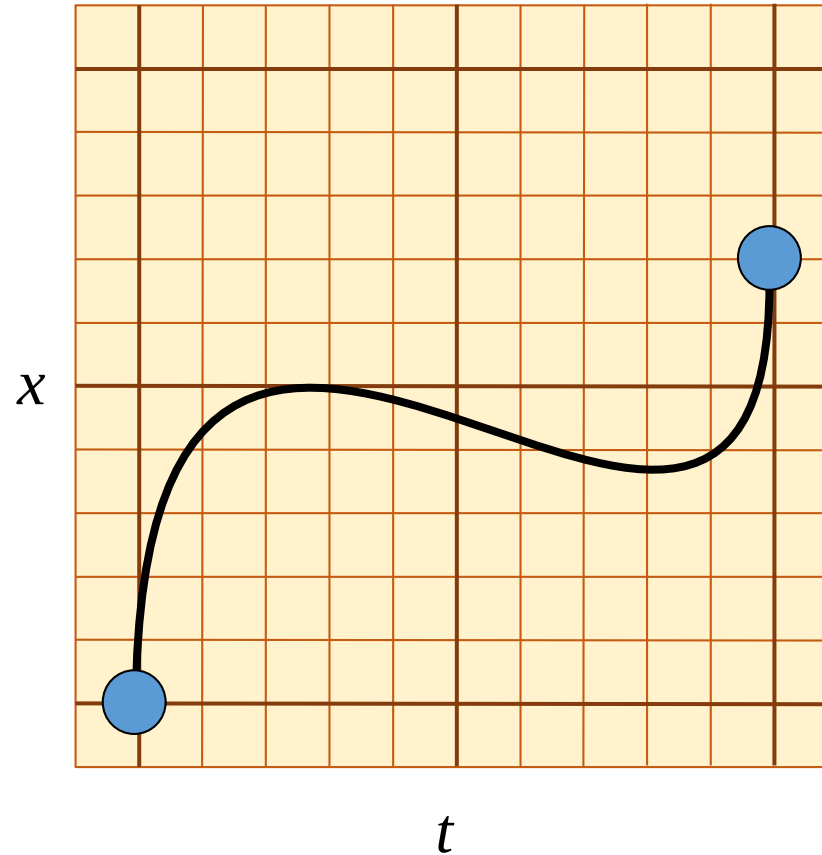
$$x(0) = x_0, x(1) = x_1$$

$$x'(0) = x'_0, x'(1) = x'_1$$

- Need cubic function

$$x(t) = At^3 + Bt^2 + Ct + D$$

$$x'(t) = 3At^2 + 2Bt + C$$



Hermite Interpolation

- Separate into coordinate functions

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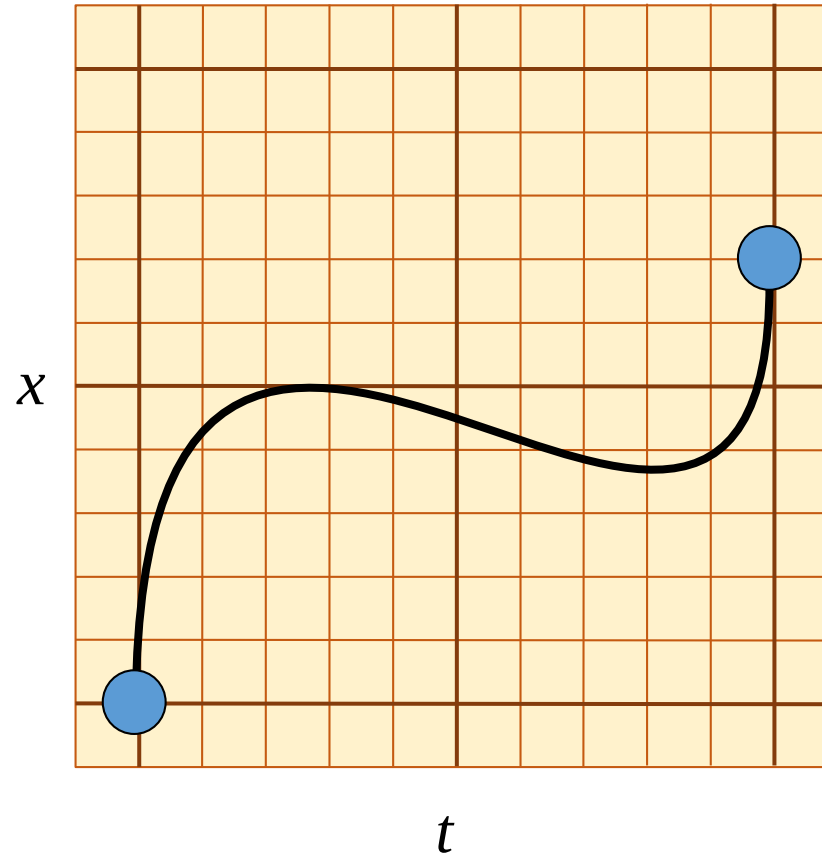
- Solve

$$A = 2x_0 - 2x_1 + x'_0 + x'_1$$

$$B = -3x_0 + 3x_1 - 2x'_0 - x'_1$$

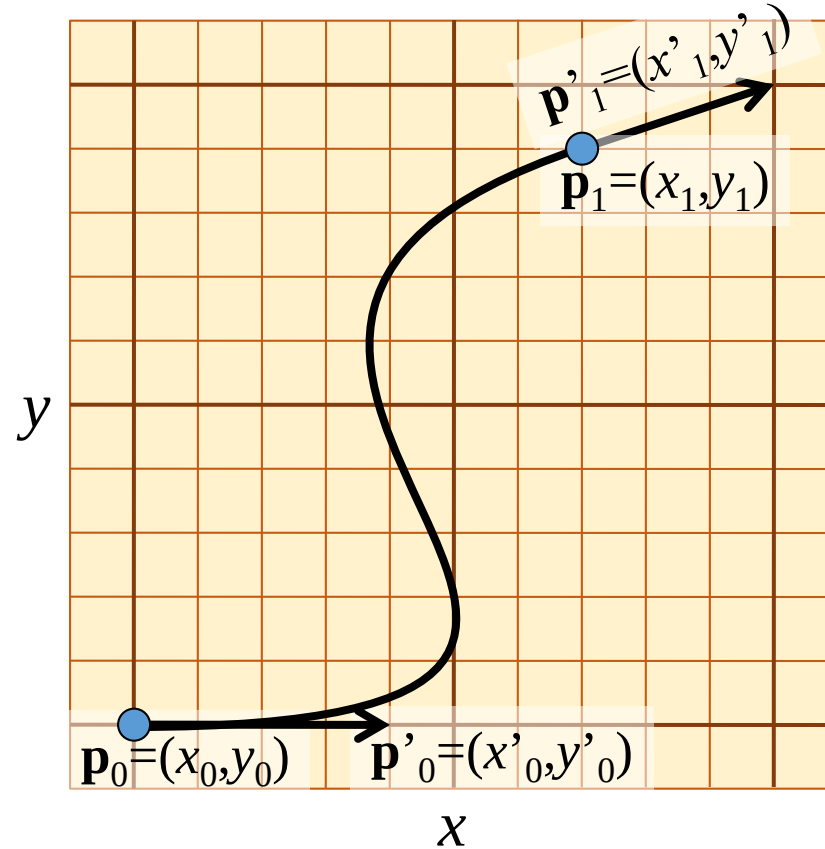
$$C = x'_0$$

$$D = x_0$$



Hermite Interpolation

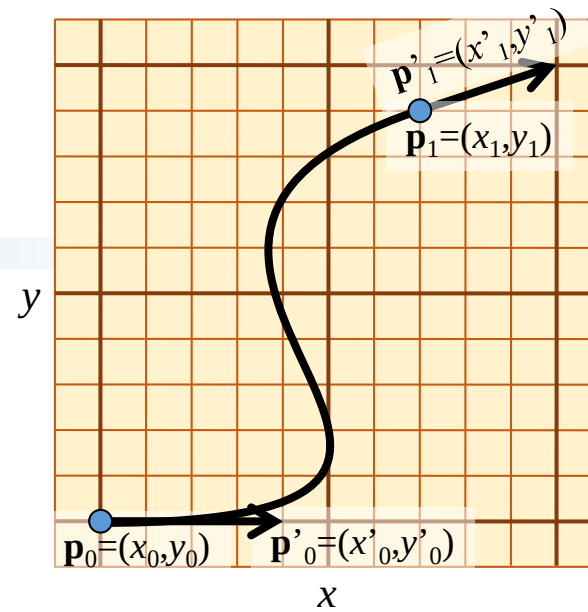
$$\mathbf{p}(t) = \begin{matrix} (2\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}'_0 + \mathbf{p}'_1) & t^3 + \\ (-3\mathbf{p}_0 + 3\mathbf{p}_1 - 2\mathbf{p}'_0 - \mathbf{p}'_1) & t^2 + \\ \mathbf{p}'_0 & t + \\ \mathbf{p}_0 & (1) \end{matrix}$$



Hermite Interpolation

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$$\mathbf{p}(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}'_0 \\ \mathbf{p}'_1 \end{bmatrix}$$

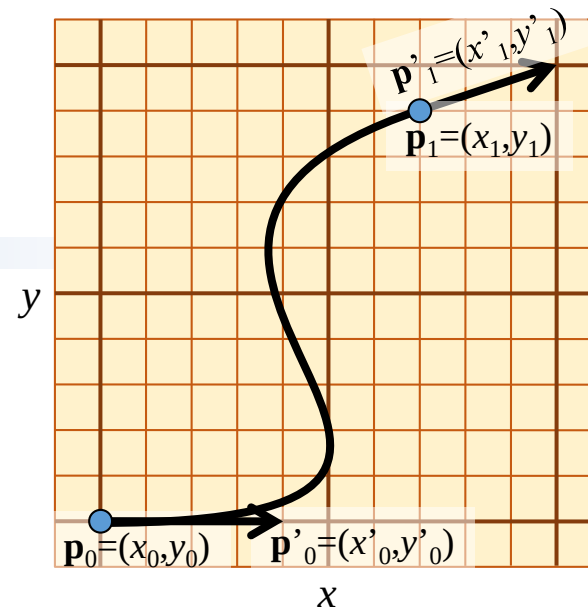


Hermite Interpolation

$$\mathbf{p}(t) = \begin{matrix} (2\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}'_0 + \mathbf{p}'_1) & t^3 + \\ (-3\mathbf{p}_0 + 3\mathbf{p}_1 - 2\mathbf{p}'_0 - \mathbf{p}'_1) & t^2 + \\ \mathbf{p}'_0 & t + \\ \mathbf{p}_0 & (1) \end{matrix}$$

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$$\mathbf{p}(t) = \begin{matrix} (2t^3 - 3t^2 + 1) & \mathbf{p}_0 + \\ (-2t^3 + 3t^2) & \mathbf{p}_1 + \\ (t^3 - 2t^2 + t) & \mathbf{p}'_0 + \\ (t^3 - t^2) & \mathbf{p}'_1 \end{matrix}$$



Hermite Interpolation

$$\mathbf{p}(t) = \begin{pmatrix} (2\mathbf{p}_0 - 2\mathbf{p}_1 + \mathbf{p}'_0 + \mathbf{p}'_1) \\ (-3\mathbf{p}_0 + 3\mathbf{p}_1 - 2\mathbf{p}'_0 - \mathbf{p}'_1) \\ \mathbf{p}'_0 \\ \mathbf{p}_0 \end{pmatrix} \begin{pmatrix} t^3 + \\ t^2 + \\ t + \\ (1) \end{pmatrix}$$

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$$\mathbf{p}(t) = \begin{pmatrix} (2t^3 - 3t^2 + 1) \mathbf{p}_0 + \\ (-2t^3 + 3t^2) \mathbf{p}_1 + \\ (t^3 - 2t^2 + t) \mathbf{p}'_0 + \\ (t^3 - t^2) \mathbf{p}'_1 \end{pmatrix}$$

